Final Report of the Minor Research Project

"Study of possibilities of Utility Functions in Economics and Commerce"

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1 Introduction

The word utility denotes the want satisfying power of a commodity or service. The consumer is assumed to be rational. Given his income and the market prices of various commodities, he plans the spending of his income so as to attain the highest possible satisfaction or utility. This is the axiom of utility maximization. Consider a consumer faced with possible consumption bundles in some set X, his consumption set. The consumer is assumed to have preferences on the consumption bundle in X; X>Y means, "the consumer thinks that the bundle X is at least as good as the bundle Y".

In economic analysis it is often convenient to summarize a consumer's behavior by means of a utility function; ie, a function U: X-> R such that X > Y iff U (X) > U (Y).If m denotes the fixed amount of money available to a consumer, and P= (p_1, p_2, …,pk), the vector of prices of goods, 1,2,…,k; the consumer's problem could be stated as

$$V(P,m) = \max U(X) \text{ such that } PX < m \qquad \text{---(1);}$$

The function V(P,m), called the indirect utility function , gives us the maximum utility achievable at given prices and income.

$$e(P,u) = \min PX \text{ such that } U(X) > u \qquad \text{---(2),}$$

called the expenditure function , gives the minimum cost of achieving a fixed level of utility u.

Considering (1) and (2) and the following assumptions

1. The utility function is continuous
2. Preferences satisfy local non satiation

3. Answers to both (1) and (2) exist , we could see that utility maximization implies expenditure minimization and expenditure minimization implies utility maximization.

4. Some of the well known utility functions are

1.CES (constant elasticity of substitution) utility function where

$U(x ,y)=(X^{\alpha}+ Y^{\beta})$, α and β being constants of elasticity

2. Cobb- Douglas utility functions, which is given by,

$U(x, y) = X^{\alpha} Y^{\beta}$

3. Money metric utility function, which gives the minimum expenditure at prices P necessary to purchase a bundle at least as good as X.

ie, min PZ such that $U(Z) > U(X)$

4.Utility function based on the principle of relative risk aversion

Consider a consumer with wealth w and suppose that he is offered gambles of the form: with probability p he will receive x% of his current wealth ; with probability (1-p) he will receive y% of his current wealth. If the consumer evaluates gambles using expected utility, the utility of this gamble will be, $pU(xw) +(1-p)U(yw)$.

Relative gambles of this sort often arise in economic problems. For eg. The return on investments is usually stated relative to the level of investment.

In general, the expected utility of a gamble depends on the entire probability distribution of the outcomes. However, in some circumstances the expected utility of a gamble will only depend on certain summary statistics of the distribution. The most common example of this is a mean- variance utility function. A useful case when mean-variance analysis is justified is the case when wealth is normally distributed.

One particular case that is of special interest is when the consumer has a utility function of the form $U(X) = -e^{-x}$. This utility function exhibits constant absolute risk aversion. More over, when wealth is normally distributed, $E[U(w)] = -e^{-w}$ ie, the expected utility is increasing in wealth. Previous discussions in the accounting literature on the use of an investment project's profit distribution have concentrated on determining the probabilities of attaining various profit levels. Assume now that one has to choose between alternative investments A and B, and the profit probability distributions are given for both investments. These profit probability distributions usually do not resolve the problem. A general theoretical solution for evaluating profit distributions of arbitrary forms has been presented by Farrar(1962). Assuming that an investor's objective is to maximize his expected utility , Farrar has shown that

$$E(U(W)) = a\,\mu_1 + b\,\mu_2 + c\,\mu_3 + \ldots + r\,\mu_r + \ldots$$

Where w=current wealth, μ = mean and μ_i = i^{th} central moment of the project's stochastic profit y.

Objective

This work aims in studying the possibilities and so the applications of this utility function in various fields, especially , in the fields related to Economics and Commerce, such as portfolio management, stock market, insurance etc.

2 Utility Theory

Origin

Jeremy Bentham a classical economist and the founder of London School of Economics, introduced the concept of 'Utility ' in economic analyses. But the classical utility approach to the theory of individual consumer demand arose in the 1870's with the contributions of william Stanley Jevons, Karl Menger and Walras. However, the utility approach to demand in the present form owes much to Alfred Marshall.

When the consumer consumes a good, he derives some benefits or satisfaction from it. Utility may be defined as the satisfaction which a consumer gets from the consumption of certain units or quantity of a commodity. In fact, the term utility is used to signify the want satisfying power of a commodity.

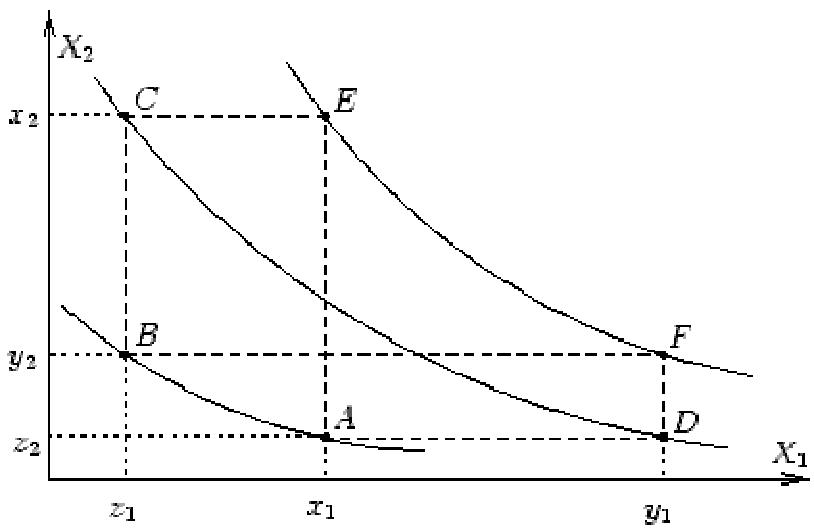
The following are the three approaches to the analysis of the 'Law of Demand' and consumer behavior based on utility.

1. Marshall's Cardinal Utility Approach
2. Hicks-Allen's Indifference curve Approach
3. Sameulson's Revealed Preference Approach

In the first approach, utility of a commodity depends upon the units or quantity of a commodity consumed. Ie, $U_X = f(q_x >_< 0)$, where U_x denotes the total utility from the consumption of x commodity and q_x signifies the quantity of x commodity consumed if there are one commodity in the consumption basket of the consumer , the utility function can be stated as follows .$U= f(x_1,x_2,\ldots,x_n)$ where U denotes the total utility if there are n commodities in the basket with quantities $x_1,x_2,\ldots,x_n$

The indifference curve approach to the analysis of demand and consumer behavior was originally developed by Parreto, slutsky and Egeworth.However, the most comprehensive exposition of the indifference curve analysis was furnished by J.R. Hicks in his book 'Value and Capital'

Hicksian indifference curve approach is based on the assumption that utility is ordinal ie a consumer can only rank the 'basket of goods' according to the satisfaction or utility of each basket . An indifference curve may be defined as the curve which represents the consumers behavior of indifference towards the various combinations of two goods giving the same level of satisfaction.



The curves passing through points *A* and *B*, *C* and *D*, and *E* and *F* are called indifference curves. They represent points in the outcome set that are all indifferent (from the Decision Maker's point of view). The first indifference illustrates that *A* and *B* belong to the same indifference curve. Similarly, the second indifference means that *C* and *D* belong to the same indifference curve.

Economists distinguish between cardinal utility and ordinal utility. When cardinal utility is used, the magnitude of utility differences is treated as an ethically or behaviorally significant quantity. On the other hand, ordinal utility captures only ranking and not strength of preferences. An important example of a cardinal utility is the probability of achieving some target.

Utility functions of both sorts assign real numbers (utils) to members of a choice set. For example, suppose a cup of coke has utility of 120 utils, a cup of tea has a utility of 80 utils, and a cup of water has a utility of 40 utils. When speaking of cardinal utility, it could be concluded that the cup of coke is better than the cup of tea by exactly the same amount by which the cup of tea is better than the cup of water. One is not entitled to conclude, however, that the cup of tea is two thirds as good as the cup of coke, because this conclusion would depend not only on magnitudes of utility differences, but also on the "zero" of utility.

It is tempting when dealing with cardinal utility to aggregate utilities across persons. The argument against this is that interpersonal comparisons of utility are suspect because there is no good way to interpret how different people value consumption bundles.

When ordinal utilities are used, differences in utils are treated as ethically or behaviorally meaningless: the utility values assigned encode a full behavioral ordering between members of a choice set, but nothing about *strength of preferences*. In the above example, it would only be possible to say that coffee is preferred to tea to water, but no more.

Neoclassical economics has largely retreated from using cardinal utility functions as the basic objects of economic analysis, in favor of considering agent preferences over choice sets. As will be seen in subsequent sections, however, preference relations can often be rationalized as utility functions satisfying a variety of useful properties.

Ordinal utility functions are equivalent up to monotone transformations, while cardinal utilities are equivalent up to positive linear transformations.

Utility functions

While preferences are the conventional foundation of microeconomics, it is often convenient to represent preferences with a utility function and reason indirectly about preferences with utility functions. Let X be the consumption set, the set of all mutually-exclusive packages the consumer could conceivably consume (such as an indifference curve map without the indifference curves). The consumer's utility function ranks each package in the consumption set. If $u(x) \geq u(y)$, then the consumer strictly prefers x to y or is indifferent between them.

For example, suppose a consumer's consumption set is X = {nothing, 1 apple, 1 orange, 1 apple and 1 orange, 2 apples, 2 oranges}, and its utility function is u(nothing) = 0, u (1 apple) = 1, u (1 orange) = 2, u (1 apple and 1 orange) = 4, u (2 apples) = 2 and u (2 oranges) = 3. Then this consumer prefers 1 orange to 1 apple, but prefers one of each to 2 oranges.

In microeconomic models, there are usually a finite set of L commodities, and a consumer may consume an arbitrary amount of each commodity. This gives a consumption set of , and each package is a vector containing the amounts of each commodity. In the previous example, we might say there are two commodities: apples and oranges. If we say apples is the first commodity, and oranges the second, then the consumption set X = and u (0, 0) = 0, u (1, 0) = 1, u (0, 1) = 2, u (1, 1) = 4, u (2, 0) = 2, u (0, 2) = 3 as before. Note that for u to be a utility function on X, it must be defined for every package in X.

In order to simplify calculations, various assumptions have been made of utility functions.

- CES (*constant elasticity of substitution*, or *isoelastic*) utility is one with constant relative risk aversion
- Exponential utility exhibits constant absolute risk aversion
- Quasilinear utility
- Homothetic utility

Most utility functions used in modeling or theory are well-behaved**.** They usually exhibit monotonicity, convexity, and global non-satiation. There are some important exceptions, however.

Expected utility

The expected utility model was first proposed by Daniel Bernoulli as a solution to the St. Petersburg paradox. Bernoulli argued that the paradox could be resolved if decision makers displayed risk aversion and argued for a logarithmic cardinal utility function.

The first important use of the expected utility theory was that of John von Neumann and Oskar Morgenstern who used the assumption of expected utility maximization in their formulation of game theory.

Additive von Neumann-Morgenstern Utility

In older definitions of utility, it makes sense to rank utilities, but not to add them together. A person can say that a new shirt is preferable to a baloney sandwich, but not that it is twenty times preferable to the sandwich.

The reason is that the utility of twenty sandwiches is not twenty times the utility of one ham sandwich, by the law of diminishing returns. So it is hard to compare the utility of the shirt with 'twenty times the utility of the sandwich'. But Von Neumann and Morgenstern suggested an unambiguous way of making a comparison like this.

Their method of comparison involves considering probabilities. If a person can choose between various randomized events (lotteries), then it is possible to *additively* compare the shirt and the sandwich. It is possible to compare *a sandwich with probability 1*, to *a shirt with probability p or nothing with probability 1-p*. By adjusting p, the point at which the sandwich becomes preferable defines the ratio of the utilities of the two options.

A notation for a *lottery* is as follows: if options A and B have probability p and 1-p in the lottery, write it as a linear combination: p U(A) + (1-p) U(B).

More generally, for a lottery with many possible options:

$P_1 * U(A_1) + P_2 * U(A_2) + \dots$

By making some reasonable assumptions about the way choices behave, von Neumann and Morgenstern showed that if an agent can choose between the lotteries, then this agent has a utility function which can be added and multiplied by real numbers, which means the utility of an arbitrary lottery can be calculated as a linear combination of the utility of its parts.

This is called *the expected utility theorem*. The required assumptions are four axioms about the properties of the agent's preference relation over 'simple lotteries', which are lotteries with just two options. Writing to mean 'A is preferred to B', the axioms are:

1. completeness: For any two simple lotteries X and Y , either $X \geq Y$, or $Y \geq X$.
2. transitivity: if $X \geq Y$ and $Y \geq Z$, then $X \geq Z$.
3. convexity/continuity (Archimedean property)
4. independence

In more formal language: A von Neumann-Morgenstern utility function is a function from choices to the real numbers:

$$U : X \longrightarrow \mathbb{R}$$

which assigns a real number to every outcome in a way that captures the agent's preferences over both simple and compound lotteries. The agent will prefer a lottery L_2 to a lottery L_1 if and only if the expected utility of L_2 is greater than the expected utility of L_1 : $E\,U(L2) > E\,U(L1)$

Repeating in category language: *u* is a morphism between the category of preferences with uncertainty and the category of reals as an additive group.

Of all the axioms, independence is the most often discarded. A variety of generalized expected utility theories have arisen, most of which drop or relax the independence axiom.

Utility of money

One of the most common uses of a utility function, especially in economics, is the utility of money. The utility function for money is a nonlinear function that is bounded and asymmetric about the origin. These properties can be derived from reasonable assumptions that are generally accepted by economists and decision theorists, especially proponents of rational choice theory. The utility function is concave in the positive region, reflecting the phenomenon of diminishing marginal utility. The boundedness reflects the fact that beyond a certain point money ceases being useful at all, as the size of any economy at any point in time is itself bounded. The asymmetry about the origin reflects the fact that gaining and losing money can have radically different implications both for individuals and businesses. The nonlinearity of the utility function for money has profound implications in decision making processes: in situations where outcomes of choices influence utility through gains or losses of money, which are the norm in most business settings, the optimal choice for a given decision depends on the possible outcomes of all other decisions in the same time-period.

3 Utility Functions in Economics and Commerce

Situations in Economics and Finance require decision making about future based on known past. The aim of decision theory is to help decision makers (DM) who face very complex problems choosing between the different possible alternatives, taking into account the consequences of each decision and the DM's preferences. In most practical situations, the difficulty in the act of taking a decision results mainly from two problems:

First, when the DM takes her decision, some data are still uncertain. For instance, in Lauritzen and Spiegelhalter (88), a doctor must determine whether her patient has a dyspnea, a breath illness, so as to give him/her the right medication. No medical device appropriate to diagnose this illness is available to the doctor. So her diagnostic can only be based on the dyspnea's symptoms. Unfortunately, these are quite similar to those of tuberculosis, lung cancer and bronchitis. A good knowledge of the patient's activities can help the doctor in her diagnostic. The doctor will have certain pieces of information available, but there will still remain some uncertainty when she diagnoses the illness.

It may also happen that available data are incomplete.

The second problem lies in the number and the complexity of the parameters (also referred to as attributes or criteria) that the DM takes into account to reach her decision. For example, in Keeney and Raiffa (93), secretary Bracamontes, of the public works ministry, must advise president Echeverria on the possible construction of a new airport for Mexico city, and especially on the best location to build it. His advice must take into account many parameters, including noise pollution, the comfort level of the neighbor populations, the evolution of air traffic, new runway construction methods, security, and so on. Here, the complexity results not only from the high number of criteria, but also from their interaction. For instance, it is rather clear that the comfort level of the neighbor populations is intimately related to the level of noise pollution.

Note that the number of criteria to be taken into account can be awfully high. For instance, in Andersen, Andreassen, and Woldbye (86), there are up to 25 relevant criteria which, of course, interact with each other. This gives some insight of how complex practical situations can be.

Modeling preference relations

We can see intuitively that a computer program that can help decision makers taking their decisions must rely on two essential components:

1. a good modelisation of the DM's preferences (in order to take decisions that comply with the DM's desiderata);
2. a good management of risks and uncertainties.

Now, what is meant by "a good modelisation of the DM's preferences"? When taking her decision, the DM has to choose between a certain number of alternatives. Each alternative will have different consequences (also referred to as outcomes). And it is obvious that, for the DM, some consequences will be more appealing than others. The DM can thus "order" the consequences according to their being more or less appealing. Modeling the DM's preferences simply amounts to finding a mathematical or computer model that represents this ordering.

For "small" problems, the ordering may be represented by a two-dimensional array: rows and columns would correspond to the possible consequences, and to each cell of the array would be assigned one of the three following values: "-1" if the outcome corresponding to the row is preferred to that of the column, "0" if the DM is indifferent between the two outcomes, and "1" if the DM prefers the outcome corresponding to the column to that of the row. Once the array is available, extracting the preferences of the decision maker from it is a very easy task. This type of representation, known as pairwise comparison, is quite similar to the arrays of figure 2 in Eckenrode (65).

Of course, when the number of outcomes is high, this modelisation is inefficient because the size of the array is much too big. In such cases, other models have to be used, the most popular of which is utility theory.

Utility theory

The principle behind utility theory is quite simple: it is to assign to each object on which the DM has preferences a real number, in such a way that the higher the number, the preferred the object. Thus, comparing objects amounts to comparing their associated numbers, which is a trivial task for a computer. The DM expresses her preferences through a set of attributes (or criteria). Each attribute can take a certain number of values (aka levels of satisfaction). For instance, when you wish to buy a car, your comparison of the different cars available on the market will certainly be based on their brand, their price, their color, their level of comfort, and so on. So each car can be represented as a tuple (price,brand,etc). The objects can thus be represented as tuples of levels of satisfaction of attributes, and modeling the DM's preferences over these objects thus amounts to evaluate a multiple argument real valued function. This is precisely what we call a utility function. In mathematical terms:

<u>Definition 1: Utility function.</u>

Let Xbe the set of objects over which the DM has preferences and $\mathbb{R}$be the set of real numbers. Let $\succsim$be the DM's preference relation, that is, $x \succsim y$ means that the DM prefers x to y or is indifferent between x and y

$f : X \to \mathbb{R}$is a utility function representing $\succsim$if and only if

for all $x, y \in X,\ x \succsim y \Leftrightarrow f(x) \geq f(y)$.

When there are multiple attributes, the sets of levels of satisfaction of which are the X_i's, *i* in {1,...,*n*}, the object set Xcan be represented as $X = X_1 \times X_2 \times \cdots X_n$. A function $f : X \to \mathbb{R}$is a utility function representing $\succsim$if and only if

for all $(x_1, \ldots, x_n), (y_1, \ldots, y_n) \in X$,
$(x_1, \ldots, x_n) \succsim (y_1, \ldots, y_n) \Leftrightarrow f(x_1, \ldots, x_n) \geq f(y_1, \ldots, y_n)$.

Utility functions are computationally very attractive because they provide easy and fast ways to extract the DM's preferences. Moreover, unlike the pairwise comparison method, in many practical situations, they do not usually consume huge amounts of memory.

Decision under certainty, risk or uncertainty

As we saw , the DM's preferences over the set of possible alternatives are related to the consequences induced by these alternatives. As an illustration, Savage (54) gives the following example: your wife is cooking an omelet. She has already broken five eggs in a plate and she asks you to complete the cooking. There remains one unbroken egg and you wonder whether you should put it in the omelet or not. Here, you have three possible alternatives:

1. you can break the egg in the plate containing the other eggs;
2. you can break it into another plate and examine it before mixing it with the five other eggs;
3. you can not use the egg.

How can we find the best suitable decision? Well, simply by examining the consequences of each decision. Thus, if the egg is good the first alternative should be better than the other ones because the omelet will be bigger, but if it is not, by choosing the first alternative we loose the five other eggs. If we choose the second alternative and the egg is good, we stain a dish that we will have to wash, and so on. By closely examining the consequences of each alternative, we should be able to select that which seems to be the most preferable.

As shown in this example, each alternative may have several consequences depending on whether the egg is good or not. In Decision Theory, these uncertain factors (here the state of the egg) are called events and, as in Probability Theory, elementary (or atomic) events are very important. They are called states of nature. To each state of nature (the egg is good or not), the choice of one of the three possible alternatives will induce a unique consequence. Thus alternatives can be represented as sets of pairs (event, consequence), which are called acts. More formally, let $\mathcal{A}$ be the set of possible alternatives, $\mathcal{X}$ be the set of all possible consequences, and $\boldsymbol{E}$ the set of states of nature. An act is a mapping from $\boldsymbol{E}$ to $\mathcal{X}$ that assigns to each state $e \in E$ a consequence in $\mathcal{X}$. Thus the constant act f corresponding to choosing the first alternative (break the egg in the plate containing the other eggs) is such that f(good egg) = "big omelet" and f(bad egg) = "lose 5 eggs".

As alternatives can be represented by acts, we can associate to the DM's preference relation over alternatives a preference relation over the set of acts (see Savage (54) or von Neumann & Morgenstern (44) for a detailed discussion on this matter). Let us denote by $\succsim_A$ this preference relation. A utility function representing $\succsim_A$ is thus a mapping U from $\mathcal{A}$ to $\mathbb{R}$ such that

$$\text{act}_1 \succsim_A \text{act}_2 \Leftrightarrow U(\text{act}_1) \geq U(\text{act}_2)$$

Of course, preferences over acts reveal both preferences over consequences (the DM will certainly prefer having a big omelet than losing five eggs) and the DM's belief in the chances that each event obtains. Thus if the DM knows that his wife is very careful about the food she keeps in her fridge, the impact of the pair (bad egg, lose 5 eggs) in the evaluation of alternative 1 will be marginal, whereas it will become important if the DM knows that his wife usually does not pay attention to this sort of things. Thus function $\boldsymbol{U}$ must take into account the plausibility of realisation of the events. Of course, this can be done only through the knowledge that the DM has of the events and not through their "true" plausibility of realisation because the decisions choosen by the DM are only based on what he/she knows. For instance, consider a decision inducing two different consequences depending on whether head or tail obtains when tossing a fair coin. You do not know that the coin is fair, but I tossed the coin thrice and you saw that two heads and one tail obtained. So your decision will be based on the fact that the chance of obtaining a head seems higher than that of having a tail, although in practice they have the same chance of realisation. Of course, different kinds of knowledge will involve different models for function U. The following three are the most important ones in Decision Theory:

- ***Decision under certainty:*** for every state of nature a given act has always the same consequence. For instance, when you decide to choose a menu rather than another one in a restaurant, consequences are known for sure: you know what you will eat (well, at least this should be). Let $\succsim_A$be the preference relations over the set of acts and $\succsim$be that over the set of consequences. Assume that $\succsim_A$and $\succsim$are represented by $U : A \rightarrow \mathbb{R}$and $f : X \rightarrow \mathbb{R}$respectively. Let x_{act}denote the consequence of "act". Then Decision under certainty amounts to:

  for all $\text{act} \in A,\ U(\text{act}) = f(x_{\text{act}})$.

- ***Decision under risk:*** An act can have several consequences, depending on the realisation of an event. Moreover, an "objective" probability distribution over the events is supposed to exist and to be known by the DM. This is the case when you decide whether to gamble or not on a game such as loto: the probability of winning is known. The model used in decision under risk is called the expected utility model and has been axiomatised by von Neumann & Morgenstern (44). As an act can be represented by pairs (event,consequence) and as there exist probabilities on events, acts can be represented by sets of pairs (consequence, probability of the consequence). These sets are usually called lotteries. Thus assume that an act corresponds to the lottery $(x^1, p_1; \ldots, x^n, p_n)$, i.e. this act has consequence $x^1$with probability p_1, $x^2$with probability $p_1$and so on. Then von Neumann-Morgenstern show that function Urepresenting preferences over acts is decomposable as:

  $$U(\text{act}) = \sum_{i=1}^{n} p_i f(x^i),$$

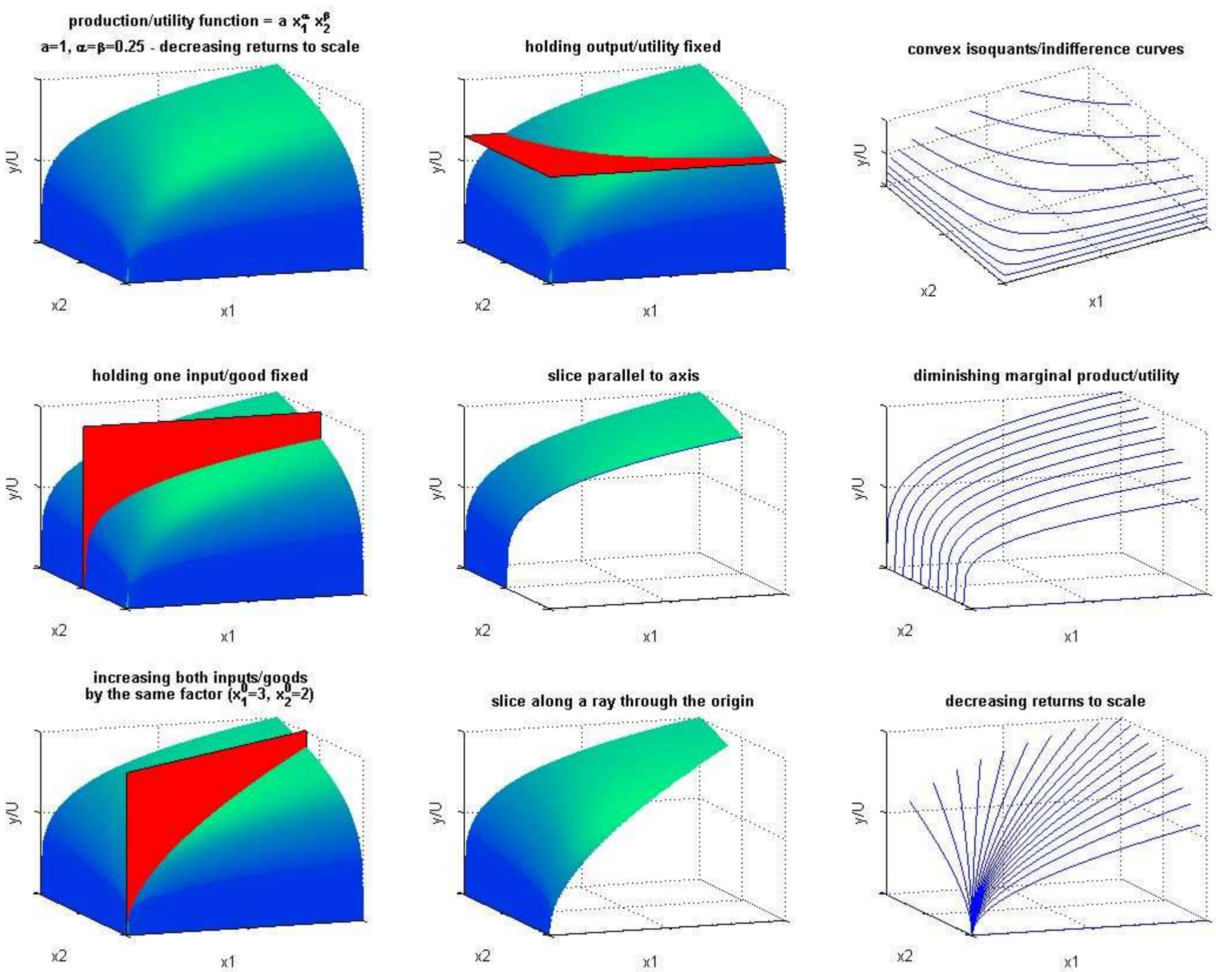
  where $f : X \rightarrow \mathbb{R}$is a utility function representing the DM's preferences over the outcomes.

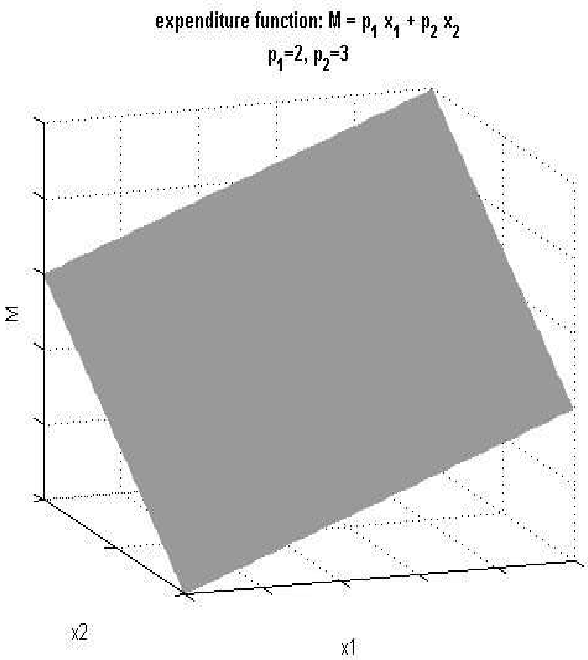
- ***Decision under uncertainty:*** this is quite similar to the preceding case except that we do not assume the existence of a probability distribution on the events set but rather this one is derived from a set of axioms (see Savage (54)) that express that fact that the DM has a "rational" behaviour. Here the probability distribution is not "objective" as in von Neumann-Morgenstern's theory but it is subjective, that is it expresses the beliefs of the DM concerning the chances of realisation of the events (instead of the "true" chance that the event will obtain). In this model, as in von Neumann-Morgenstern, the utility U of an act is decomposable as:

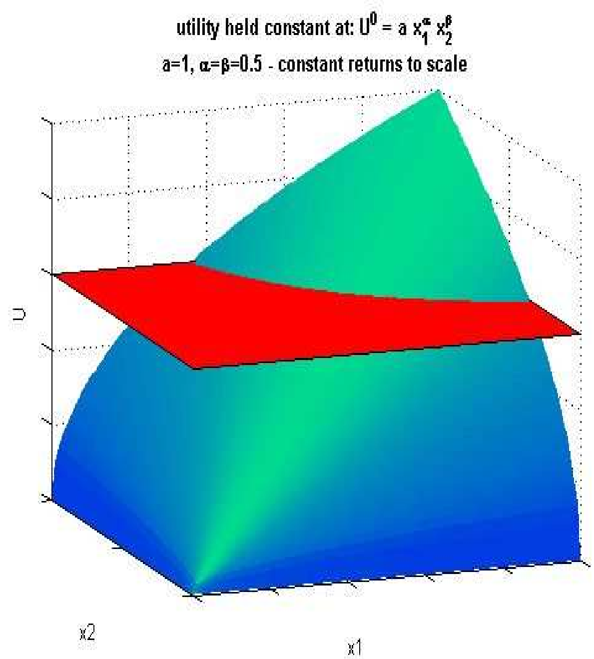
$$U(\text{act}) = \sum_{i=1}^{n} p_i f(x^i),$$

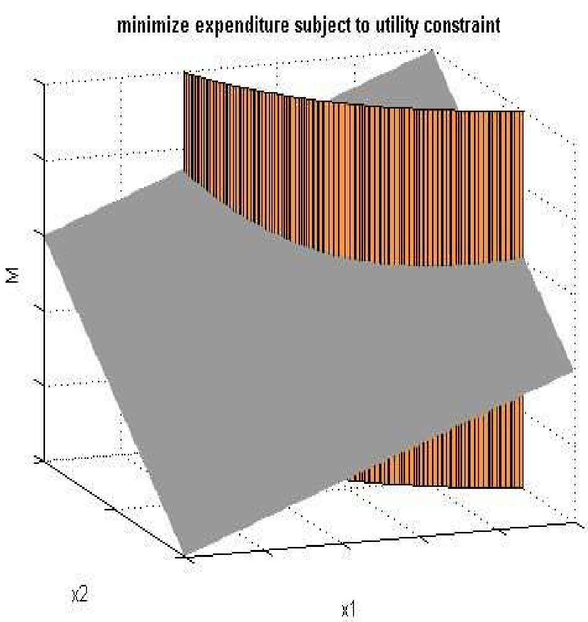
where $f : X \to \mathbb{R}$ is a utility function representing the DM's preferences over the outcomes, and p_i is the subjective probability that the DM assigns to event x^i.

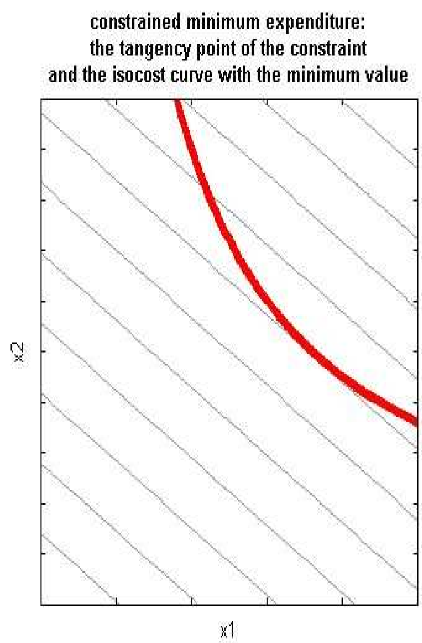
4 Cobb-Douglas Production/Utility functions in Three dimensions

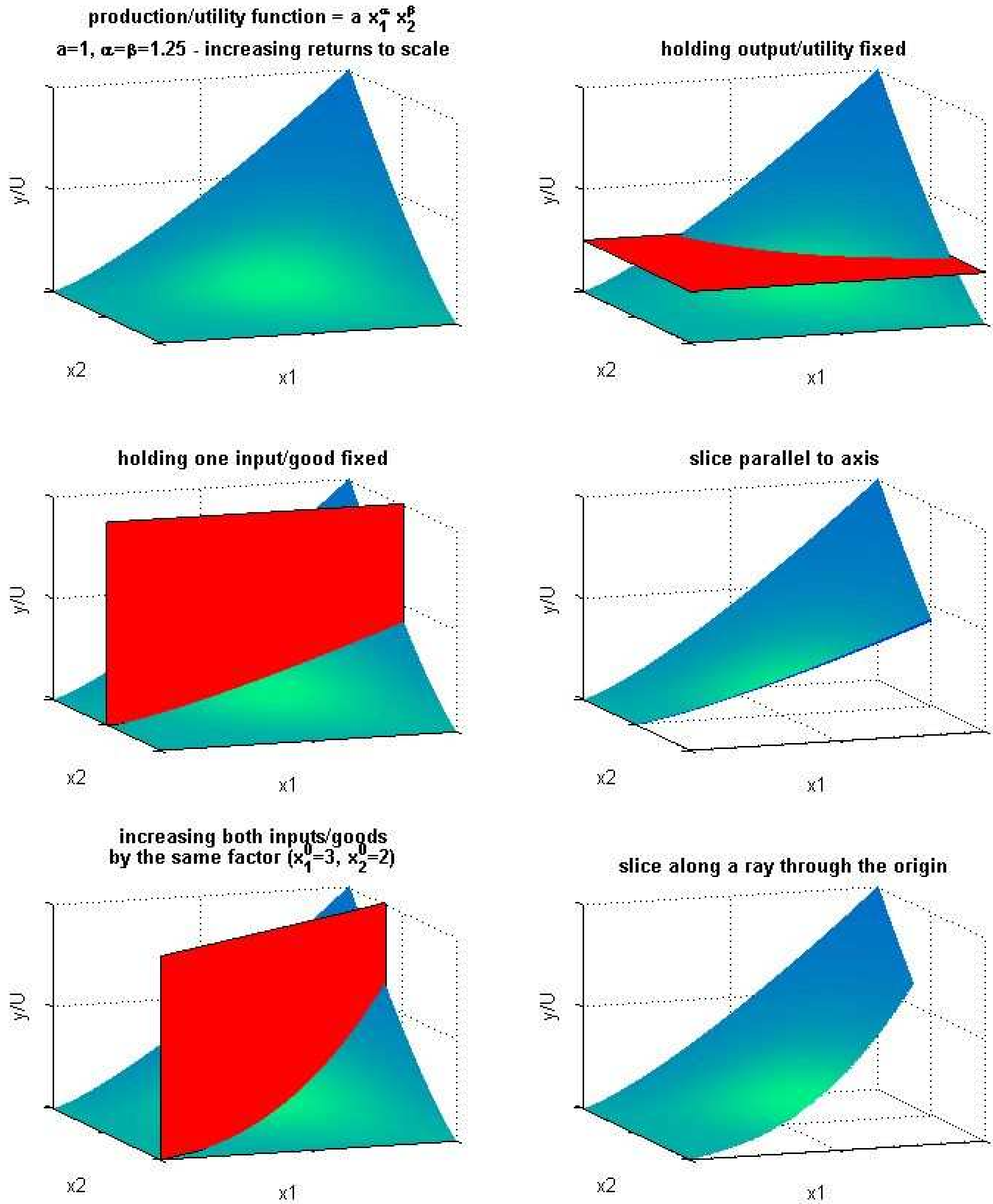



expenditure function: $M = p_1 x_1 + p_2 x_2$
$p_1=2, p_2=3$
M
x2
x1


utility held constant at: $U^0 = a x_1^\alpha x_2^\beta$
$a=1, \alpha=\beta=0.5$ - constant returns to scale
U
x2
x1


minimize expenditure subject to utility constraint
M
x2
x1


constrained minimum expenditure:
the tangency point of the constraint
and the isocost curve with the minimum value
x2
x1


production/utility function = a x_1^{α} x_2^{β}
a=1, α=β=1.25 - increasing returns to scale
y/U
x2
x1
holding output/utility fixed
y/U
x2
x1
holding one input/good fixed
y/U
x2
x1
slice parallel to axis
y/U
x2
x1
increasing both inputs/goods
by the same factor (x_1^0=3, x_2^0=2)
y/U
x2
x1
slice along a ray through the origin
y/U
x2
x1

5 Conclusion

Utility, though**,** considered by many as a psychological phenomenon, utility functions, through it's association with production –expenditure functions, demand-supply functions, play key role in Economics and through it's association with expected returns and minimization of risks involved, plays a key role in decision making in financial, portfolio managements. In understanding the risks of various shares in stocks and in calculating premiums with Life Insurance Policies, one of the widely used methods is the application of Utility Theory. Though lots of work is going on in this field, it still remains to be explored thouroughly.

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