SUMMARY OF THE REPORT OF
MINOR RESEARCH PROJECT UNDER XIIth PLAN
No. F. MRP(S)-239/12-13/KLKA020/UGC-SWRO

TOPOLOGY ON GRAPHS AND HYPERGRAPHS

A graph G =(V, E) is called a Bitopological graph if there exist a set X and a set-indexer f with respect to X such that both f(V) and $f^{\oplus}(E) \cup \{\phi\}$ are topologies on X. The corresponding set-indexer is called a Bitopological set-indexer of G whenever it exists.

A bitopological set-indexer of G =(V, E) is called discrete bitopological set-indexer(denoted by $\delta -$ bitopological) if $f^{\oplus}(E) \cup \{\phi\}$ is the discrete topology on X.

δ- Bitopological graphs are particular types of Bitopological graphs. Different classes of δ- Bitopological graphs are characterized. The result showed that the complete graph K_n is δ- Bitopological if and only if $n \leq 3$. The maximal δ- Bitopological subgraphs of K_n for n = 4,5,6,7,8,9 and 10 are identified. It was also found out that the complete bipartite graph $K_{m,n}$ is δ- Bitopological with respect to set of cardinality p if and only if it is star with $2^p - 1$ edges. It was found out that the number of vertices in a δ-Bitopological graph is of the form $2^m + 2^{n-m} - 1$ where $1 \leq m \leq n$. Also the cycle C_n is δ- Bitopological if and only if n = 3. It was also proved that every star can be embedded as an induced subgraph of a δ- Bitopological graph.

Given any bitopological labelling f of a graph G with ground set X, construct a set-indexed digraph $(\vec{G}, f)$ by taking V(G) = V($\vec{G}$) and a line directed from u to v if $|f(u)| \geq |f(v)|$ with $g_f(u,v)$ = f(u)-f(v). $\vec{G}$ is ς- bitopological if $g_f(A) \cup \{\phi\}$ is a topology on X where A is the arc set of $\vec{G}$. Directed graphs using bitopological labelling were constructed. The digraphs obtained from bitopological labelling of different classes of graphs need not be transitive. If we have given a transitive digraph then it has a bitopological realization if and only if the underlying undirected graph is bitopological.

Bitopological index $\beta_\tau(G)$ of a finite graph G is the minimum cardinality of a set X such that G is bitopological with respect to X. It was proved that bitopological index of path $P_n \leq n-1$. It is a tedious task to find the bitopological index of an arbitrary tree. It may even an NP- complete problem. However we find the bitopological index $\beta_\tau(G)$ of the classes of trees with order upto six and diameter less than or equal to five. It was found that bitopological index of a uniform binary tree with one pendant edge added at the root vertex and having n levels is n.

Hypergraphs corresponding to bitopological graphs were constructed and studied the characteristics of them. The stability number of the hypergraph $H_{[P_n,f]}$ is $\left\lceil \frac{n}{2} \right\rceil$. The transversal number of $H_{[P_n,f]}$ is 1. All $H_{[P_n,f]}$ satisfy the Helly property. No $H_{[P_n,f]}$ is hereditary. Hypergraph $H_{[K_{m,n},\ f]}$ has stability number m+n-2, Transversal number 1. This hypergraph satisfies the Helly property but it is not hereditary. The hyperedges of the optimal hypergraph $ $H_{[K_3,\ f]}$ are {1}, {2} and {1, 2}. Stability number of this hypergraph is 1 and transversal number is 2. It satisfies the hereditary property but not the Helly property. It was observed that the hypergraph corresponding to the δ-bitopological labelling is hereditary but it does not satisfy the Helly property. Stability number of this hypergraph is 1 and transversal number is $|X|$.